CSE 291 – AI Agents Classical Control, Pre-deep Learning

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Thanks to David Silver's DeepMind RL Course and Sheila McIllraith's Planning Course at UofT. Some slides were adapted form there.

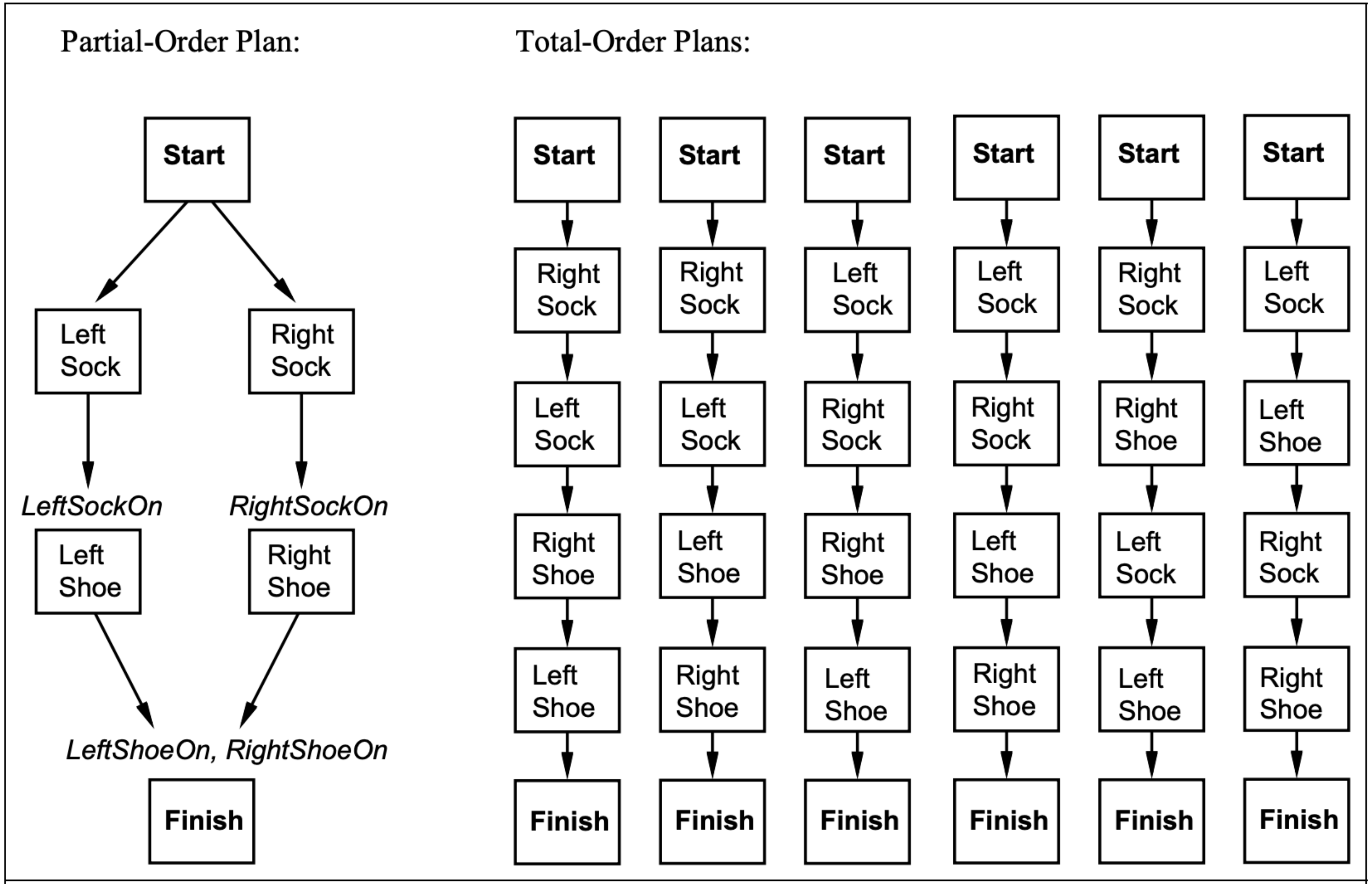
Forward Search

- Some deterministic implementations of forward search:
  - breadth-first search
  - depth-first search
  - best-first search (e.g., A*)
  - greedy search
- Breadth-first and best-first search are sound and complete But they usually aren't practical, requiring too much memory
  - Memory requirement is exponential in the length of the solution
- In practice, more likely to use depth-first search or greedy search
  - Worst-case memory requirement is linear in the length of the solution
  - In general, sound but not complete
  - But classical planning has only finitely many states
  - Thus, can make depth-first search complete by doing loop-checking

Backward Search

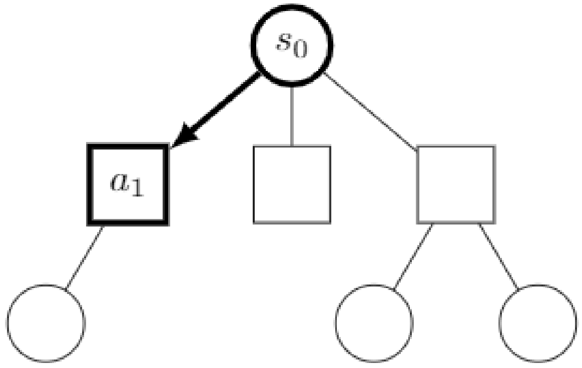
- For forward search, we started at the initial state and computed state transitions
  - new state = T(s,a)
- For backward search, we start at the goal and compute inverse state transitions
  - new set of subgoals = T^{-1}(g,a)
- To define T^{-1}(g,a), must first define relevance: An action a is relevant for a goal g if
  - a makes at least one of g's literals true, $g \cap effects(a) \neq \emptyset$
  - a does not make any of g's literals false, $g^{+} \cap effects^{-}(a) = \emptyset$ and $g^{-} \cap effects^{+}(a) = \emptyset$

Total Order and Partial Order Plans



Monte Carlo Tree Search

- 4 phases of building out and simulating paths along a search tree
- Various forms of this used in everything from Alpha Zero to modern LLM inference
- For arbitrary problem with start state s_0 and actions a_i
- All states have attributes:
  - Total simulation reward Q(s) and
  - Total no. of visits N(s)



Improvements to MCTS Components

- Improvements are possible for each of the parts I talked about
- Think about that it would take to improve selection / expansion phases

Upper Confidence Trees (UCT)

- A way of improving the selection phase by treating selection as a multi-arm bandit problem: which possible action to select that maximizes the possible payout (reward) in the future

$$\mathrm{UCT}(v_i, v) = \frac{Q(v_i)}{N(v_i)} + c\sqrt{\frac{\ln N(v)}{N(v_i)}}$$

Upper Confidence Trees (UCT)

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Exploit

Upper Confidence Trees (UCT)

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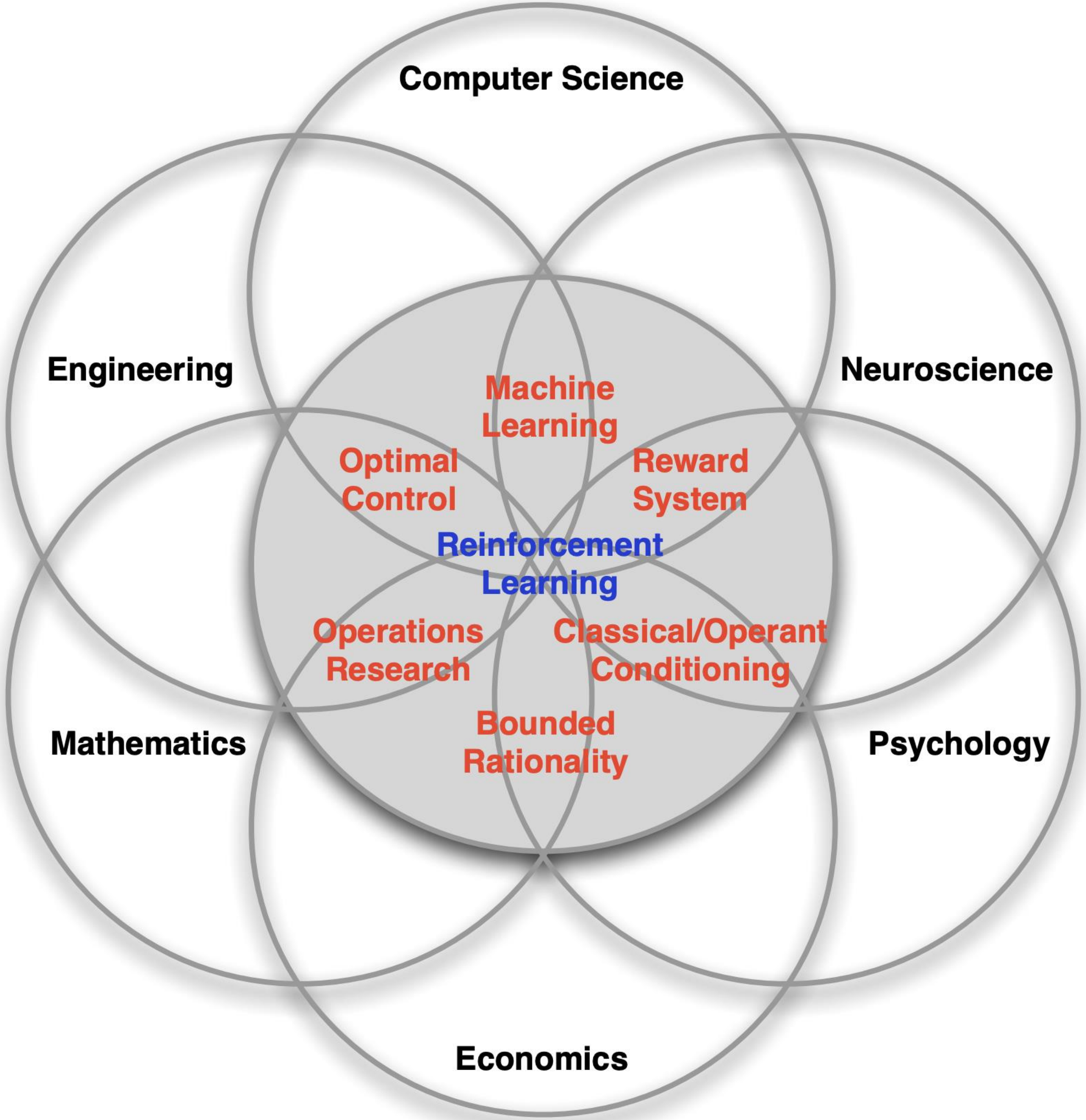
$$\mathrm{UCT}(v_i, v) = \frac{Q(v_i)}{N(v_i)} + \boxed{c\sqrt{\frac{\ln N(v)}{N(v_i)}}}$$

Exploit Explore

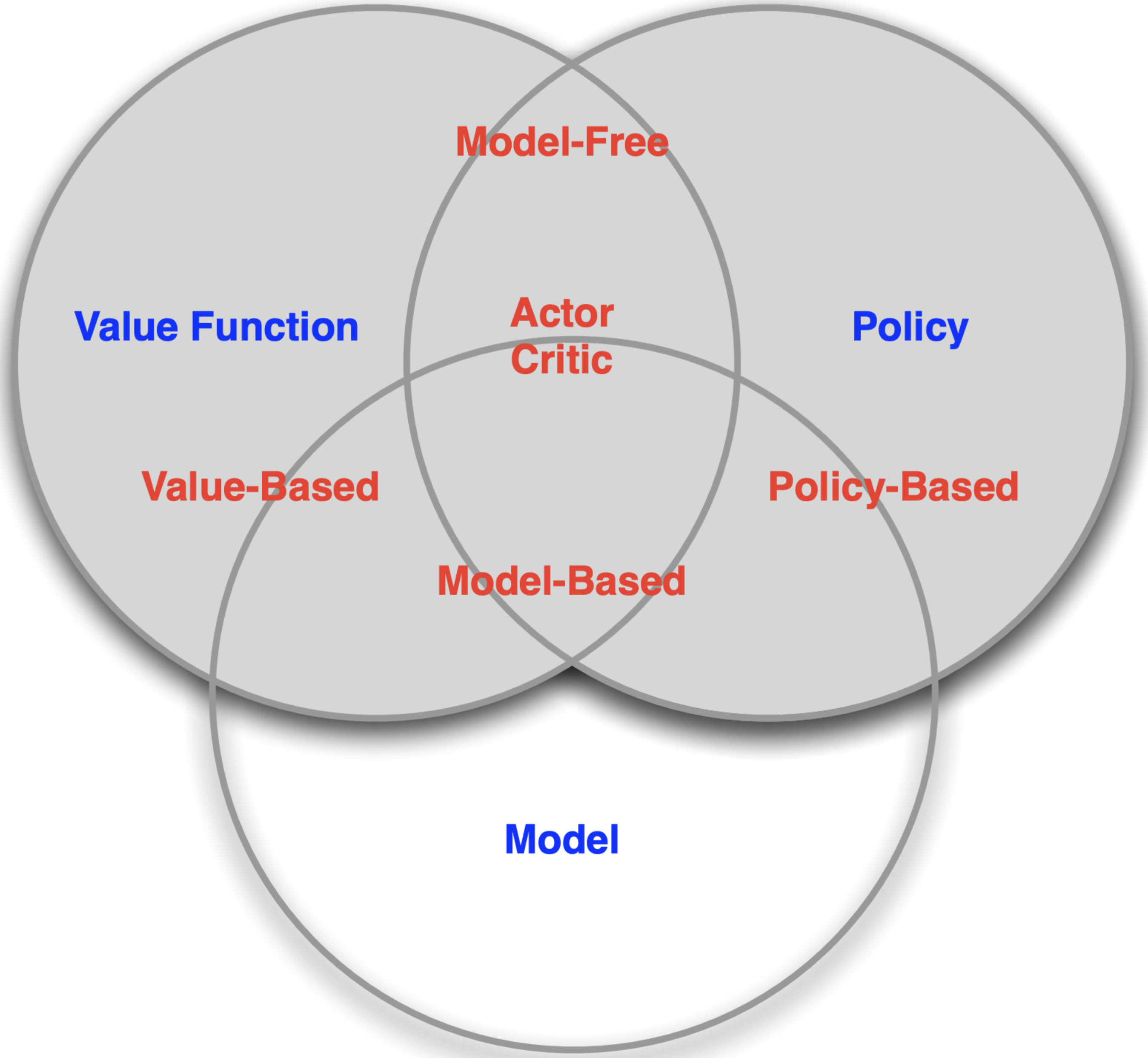
Why Reinforcement Learning?

- Reinforcement Learning:
    - The environment is initially unknown
    - The agent interacts with the environment
    - The agent improves its policy
- Planning:
    - A model of the environment is known
    - The agent performs computations with its model (without any external interaction)
    - The agent improves its policy a.k.a. deliberation, reasoning, introspection, pondering, thought, search

Origins of RL



RL Agent Taxonomy



Terminology

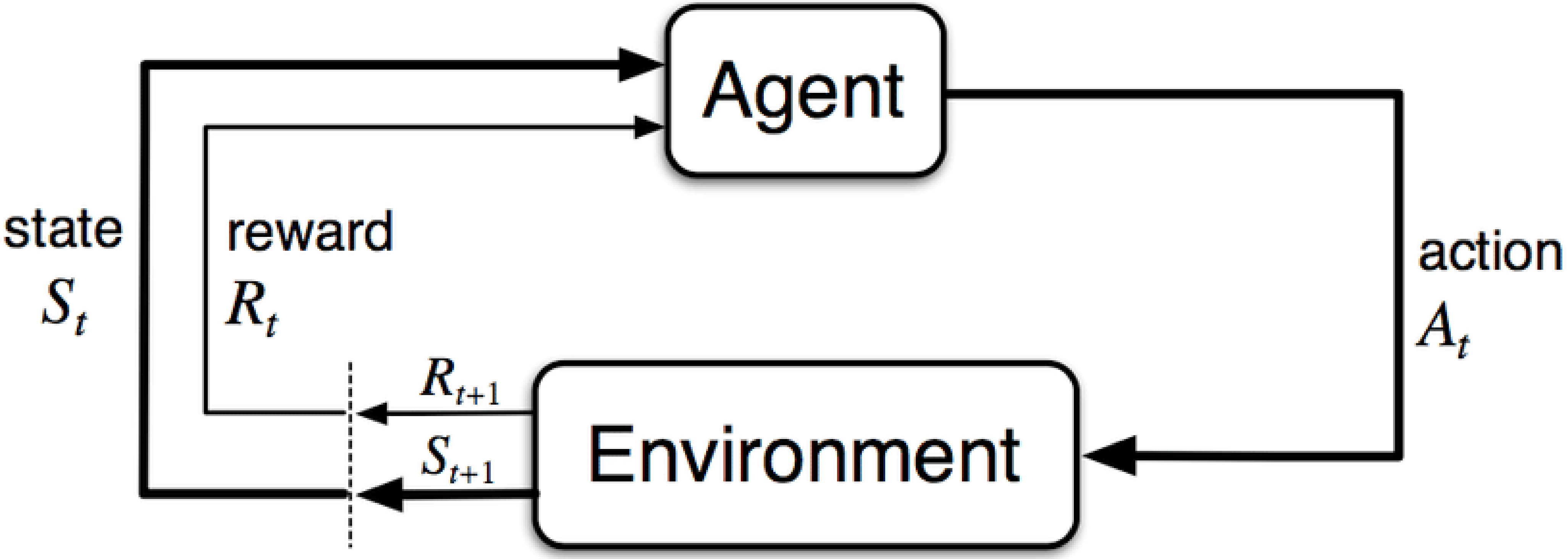
- Policy: agent’s behavior function
  - Finding optimal policy known as the <u>control</u> problem
- Value function: how good is each state and/or action
  - Finding optimal value function is known as the <u>prediction</u> problem
- Model: agent’s representation of the environment

More Terminology on Types of RL

- Model free ← will build up to today
- Model based

- On Policy ← will build up to today
  - Learn directly from your experiences “on the job”
- Off policy
  - Learn from someone else’s behavior

Markov Decision Process



Formal MDP Definition

A Markov Decision Process is a tuple <S, A,T, R, γ>

- S is a finite set of states
- A is a finite set of actions
- T is a state transition probability matrix, $T^a_{ss'} = P [S_{t+1} = s' \mid S_t = s, A_t = a]$
- R is a reward function, $R^a_s = E [R_{t+1} \mid S_t = s, A_t = a]$
- γ is a discount factor $\gamma \in [0, 1]$.

Returns and Discounting

- The return G_t is the total discounted reward from time-step t.

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- The value of receiving reward R after k + 1 time-steps is $\gamma^k R$
- γ~=0 is “myopic”, γ~=1 is “far-sighted”
- Why discount?
  - Mathematically convenient, avoids infinite returns
  - Animal/human/investment banker’s behavior shows preference for immediate reward

Formal Definition of Policy

- Distribution of action over states: $\pi(a|s) = P\ [A_t = a \mid S_t = s]$
- Policy depends only on current state not history, this is the Markov property bit of MDP (how do people get around this for cases where history does matter)
- Theorem (abridged): There always exists an optimal policy for a given finite MDP. It follow the optimal value function.

Formal Definition of Value Function

- State value: expected return starting from state s, and then following policy π
  - $v_\pi(s) = E_\pi [G_t \mid S_t = s]$
- Action value: is the expected return starting from state s, taking action a, and then following policy π
  - $q_\pi(s, a) = E_\pi [G_t \mid S_t = s, A_t = a]$

Dynamic Programming

- Building up to RL first requires understanding Dynamic Programming
- Dynamic sequential or temporal component to the problem
  Programming optimizing a “program”, i.e. a policy
- A method for solving complex problems by breaking them down into subproblems
  - Solve the subproblems → Combine solutions to subproblems

When to use DP

Dynamic Programming is a very general solution method for problems which have two properties:

- Optimal substructure:
  - Principle of optimality applies
  - Optimal solution can be decomposed into subproblems
- Overlapping subproblems:
  - Subproblems recur many times
  - Solutions can be cached and reused
- Markov decision processes satisfy both properties Bellman equation gives recursive decomposition Value function stores and reuses solutions

Prediction vs Control

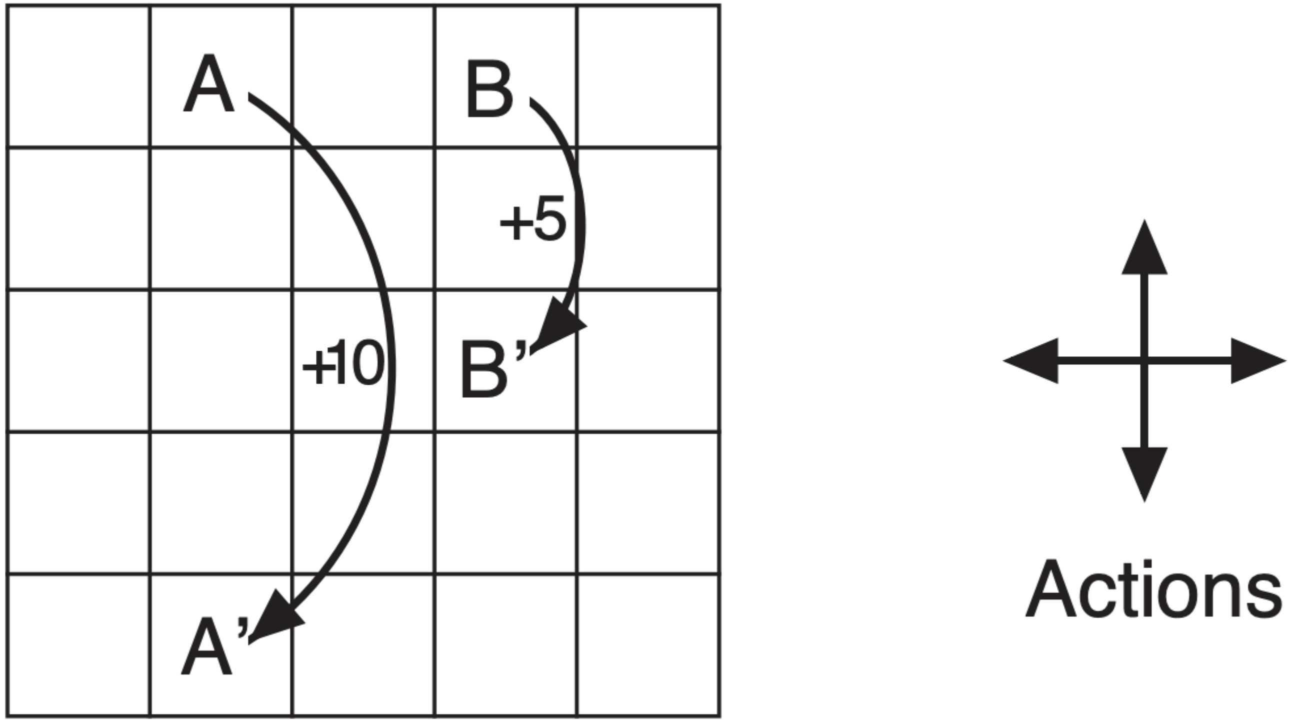
Two problems in RL

- Prediction is the problem of evaluating how good any given state is for getting rewards given a policy
- Control is the problem of selecting actions that give you a policy that maximizes reward

Planning via DP

- Dynamic programming assumes full knowledge of the MDP
- It is used for planning in an MDP
- For prediction:
  - Input: MDP <S, A,T, R, γ> and policy π
  - Output: value function v_π
- For control:
  - Input: MDP <S, A,T, R, γ>
  - Output: optimal value function v_* and: optimal policy π_*

Prediction Example



(a)

3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

(b)

Bellman Expectation

- The state-value function can again be decomposed into immediate reward plus discounted value of successor state,

  $$v_\pi(s) = E_\pi [R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s]$$

- The action-value function can similarly be decomposed,

  $$q_\pi(s, a) = E_\pi [R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a]$$

- No closed form solution (in general)

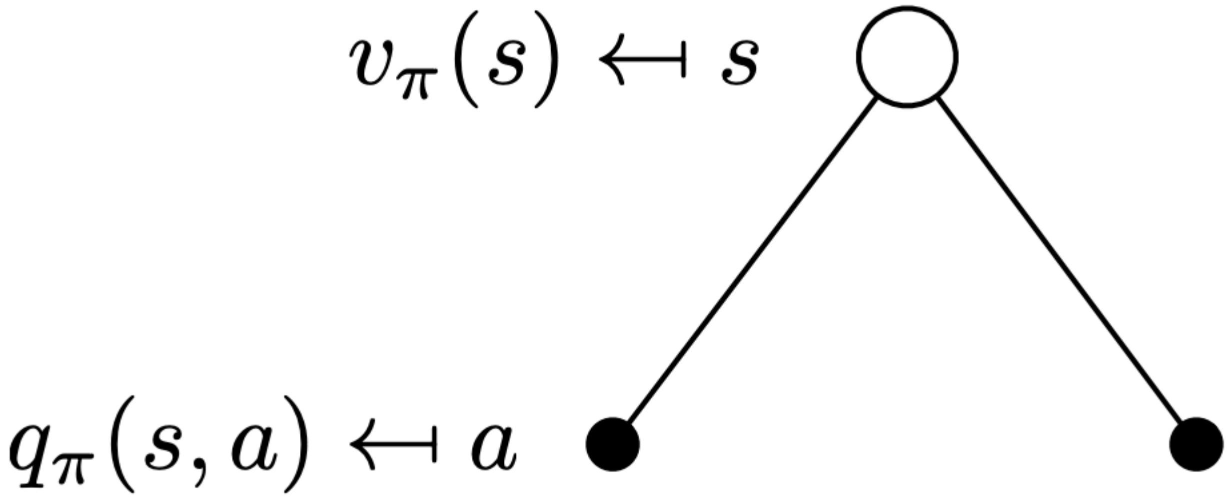
Policy Evaluation

- Problem: evaluate a given policy π
- Solution: iterative application of Bellman expectation backup

  $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_\pi$
- Using synchronous backups,
  - At each iteration k + 1
  - For all states $s \in S$ Update $v_{k+1}(s)$ from $v_k(s')$, where s' is a successor state of s

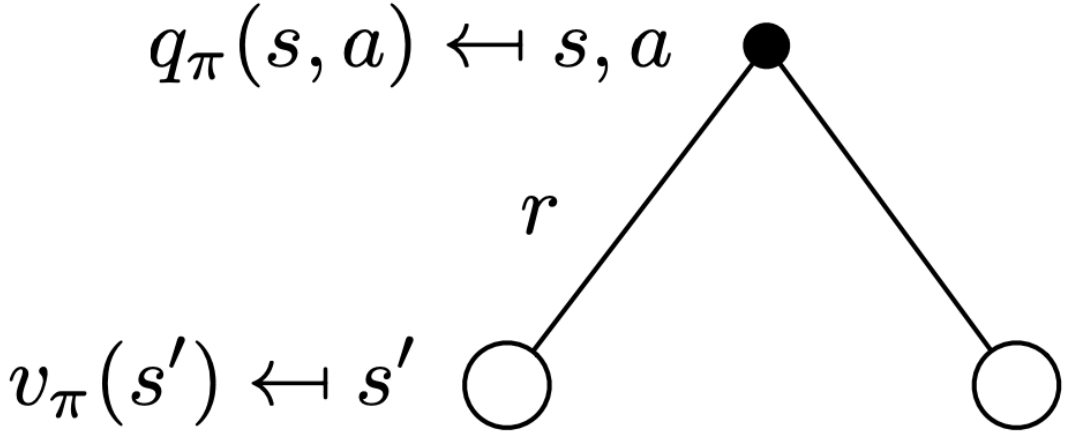
Policy Evaluation

$v_\pi(s) = \sum_{a \in A} \pi(a|s)\, q_\pi(s,a)$



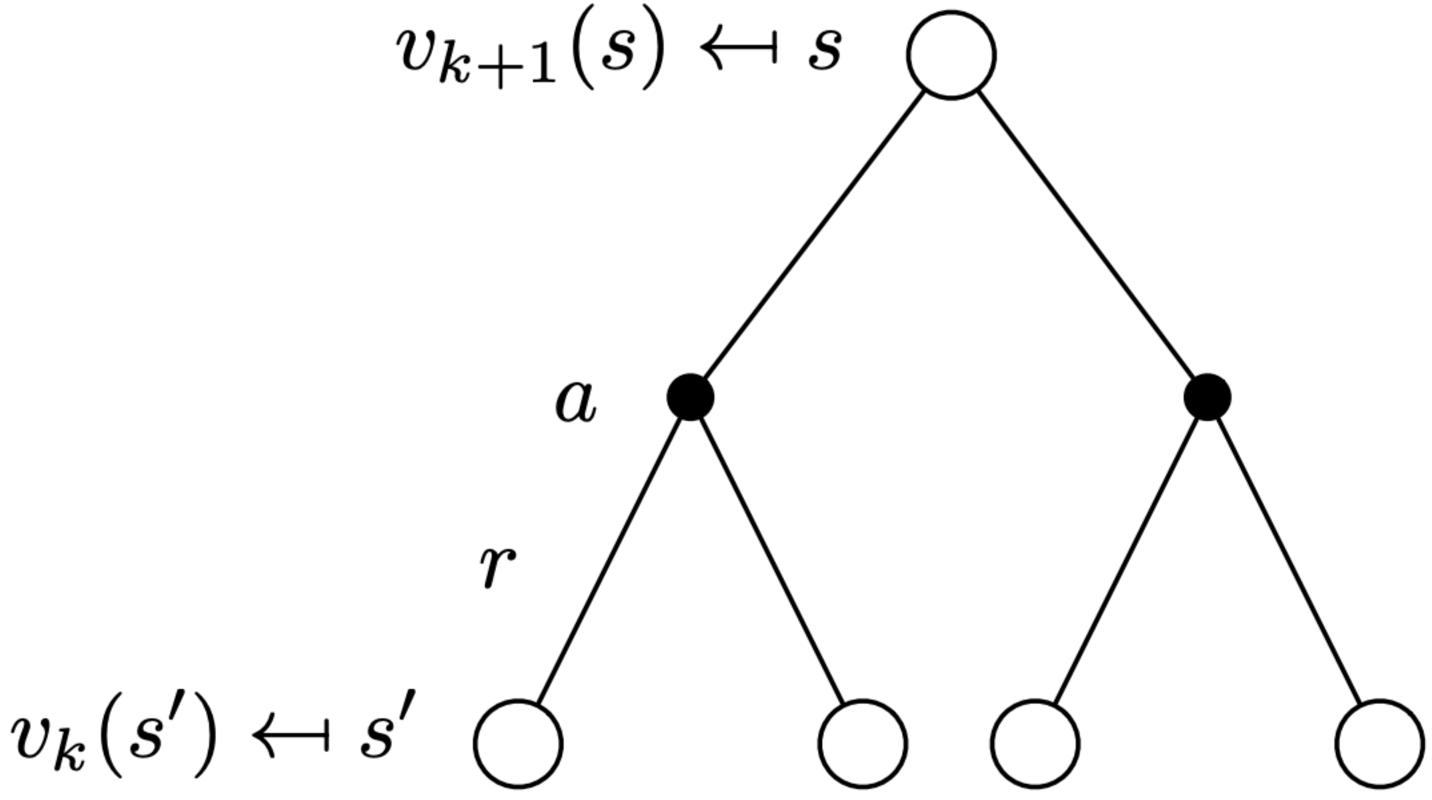
Policy Evaluation

$q_\pi(s,a) = R^a_s + \gamma \sum_{s' \in S} T^a_{ss'} v_k(s')$

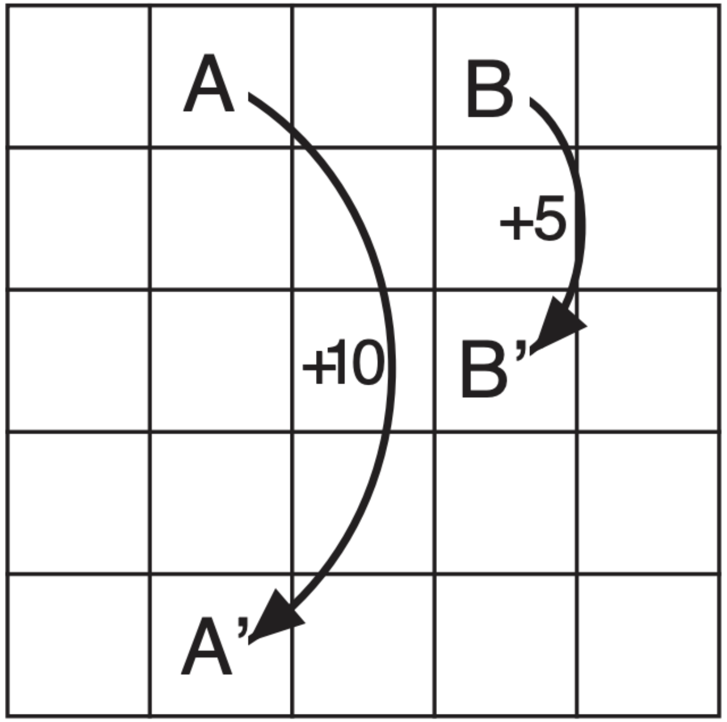


Policy Evaluation

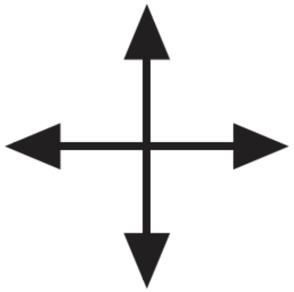
$v_{k+1}(s) = \sum_{a \in A} \pi(a|s) \left(R^a_s + \gamma \sum_{s' \in S} T^a_{ss'} v_k(s')\right)$



Prediction Example



(a)

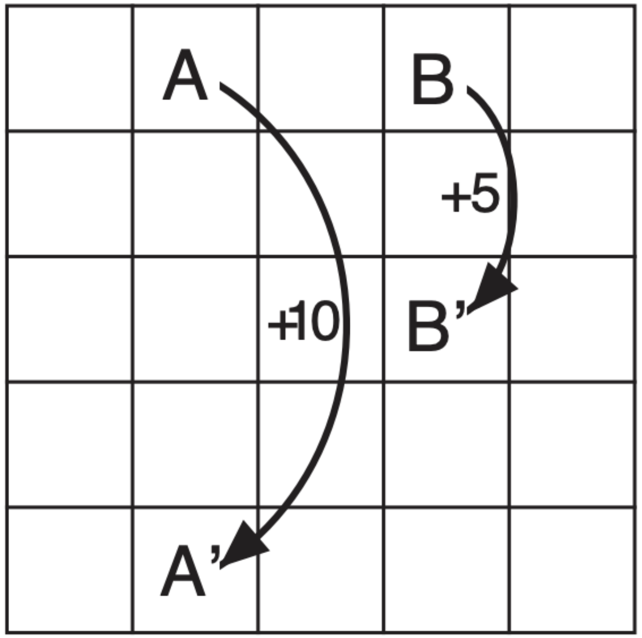


Actions

3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

(b)

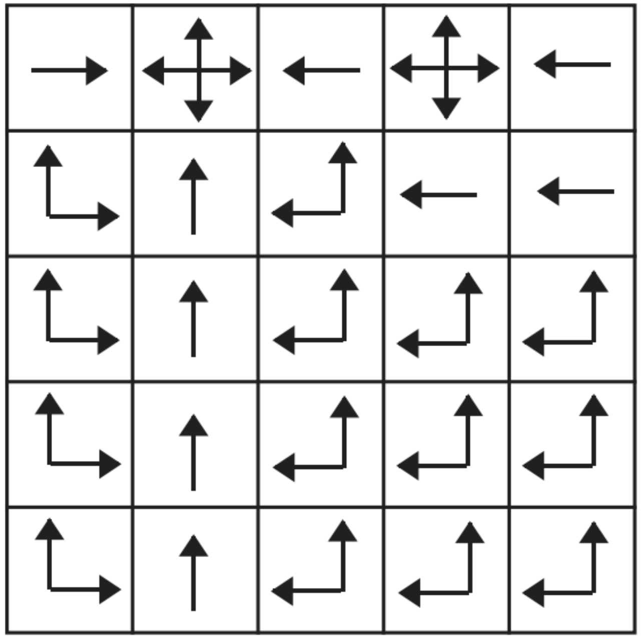
Control Example



a) gridworld

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

b) v_*



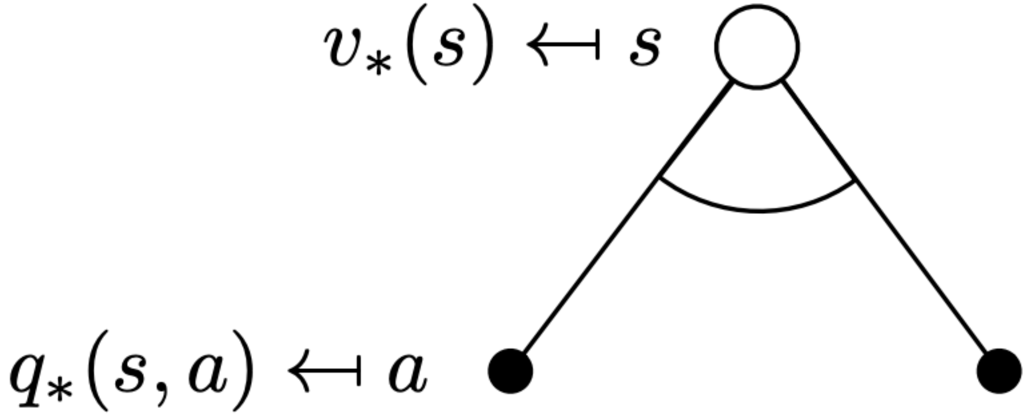
c) π_*

Bellman Optimality Equation

- Optimal state value: $v^*(s) = \max_{\pi} v_{\pi}(s)$
- Optimal action value: $q^*(s,a) = \max_{\pi} q_{\pi}(s,a)$
- Optimal policy: $\pi^*(s) = \operatorname{argmax}_{a} q^*(s,a)$

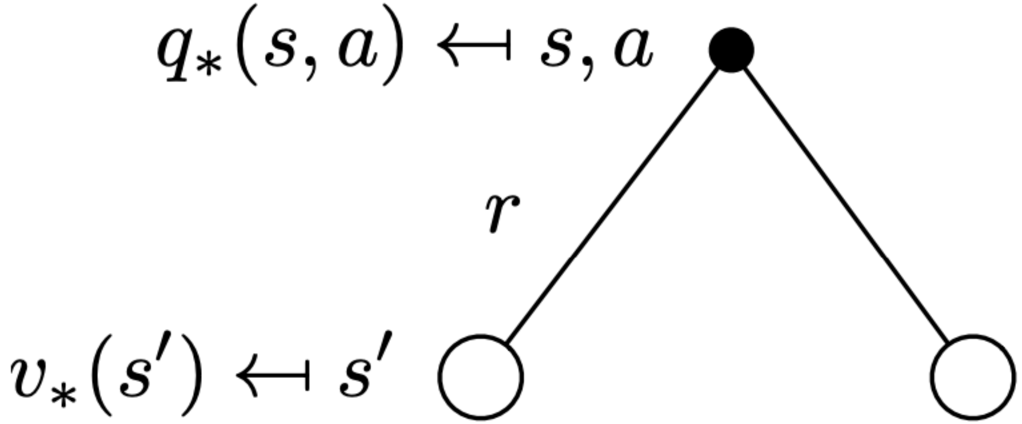
Bellman Optimality Equation

$v^*(s) = \max_a q*(s',a')$



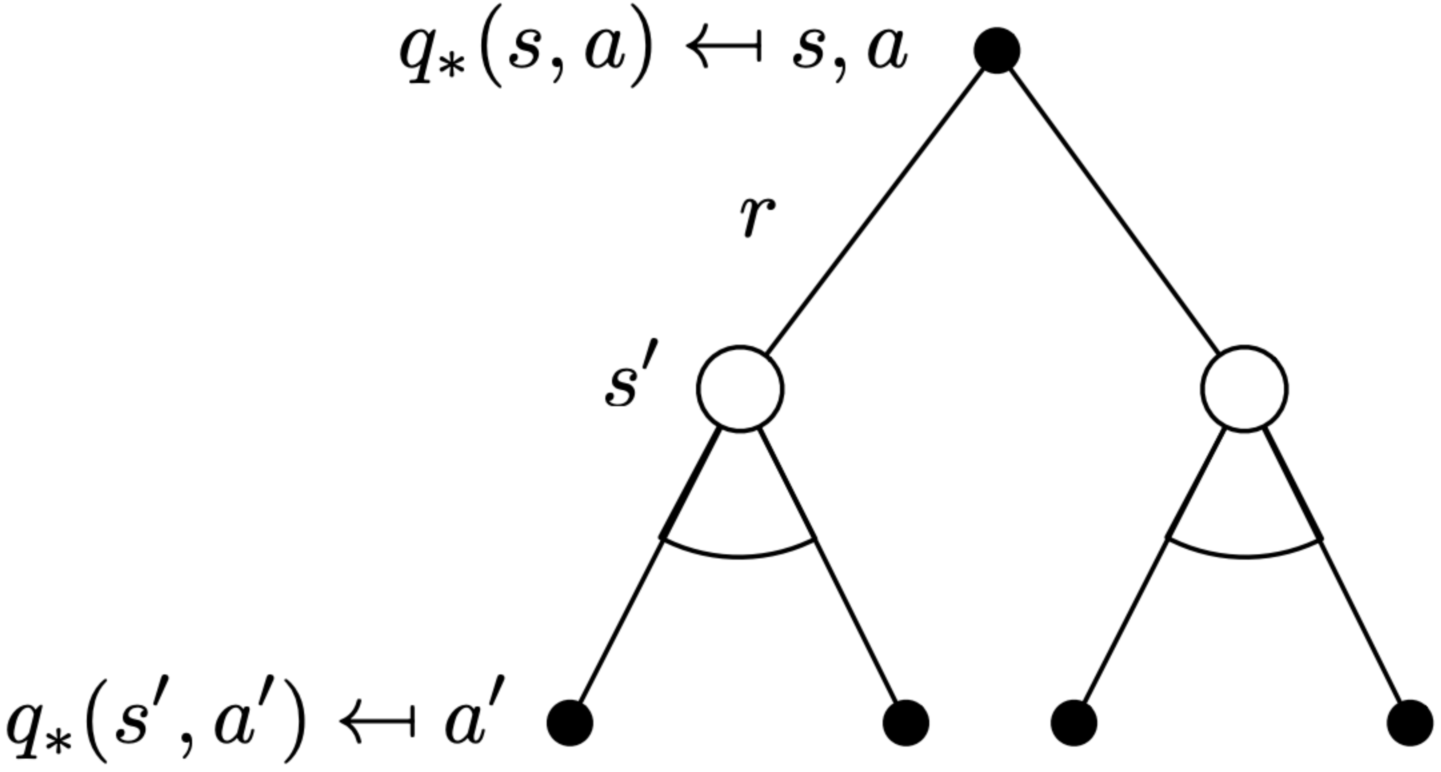
Bellman Optimality Equation

$q*(s,a) = R^a_s + \gamma \sum_{s' \in S} T^a_{ss'} v*(s')$



Bellman Optimality Equation

$q^*(s,a) = R^a_s + \gamma \sum_{s' \in S} T^a_{ss'} \max_a q^*(s',a')$



Bellman Optimality Equation

- Optimal state value: $v_*(s) = \max_{\pi} v_{\pi}(s)$
- Optimal action value: $q_*(s,a) = \max_{\pi} q_{\pi}(s,a)$
- Optimal policy: $\pi^*(s) = \text{argmax}_{a}\, q^*(s,a)$

- $q^*(s,a) = R^a_s + \gamma \sum_{s' \in S} T^a_{ss'} \max_a q_*(s',a')$
- $v^*(s) = \max_{a \in A} q^*(s, a)$

Policy Iteration

Given a policy π

- Evaluate the policy π $v_\pi(s) = E\ [R_{t+1} + \gamma R_{t+2} + \ldots | S_t = s]$
- Improve the policy by acting greedily with respect to v_π

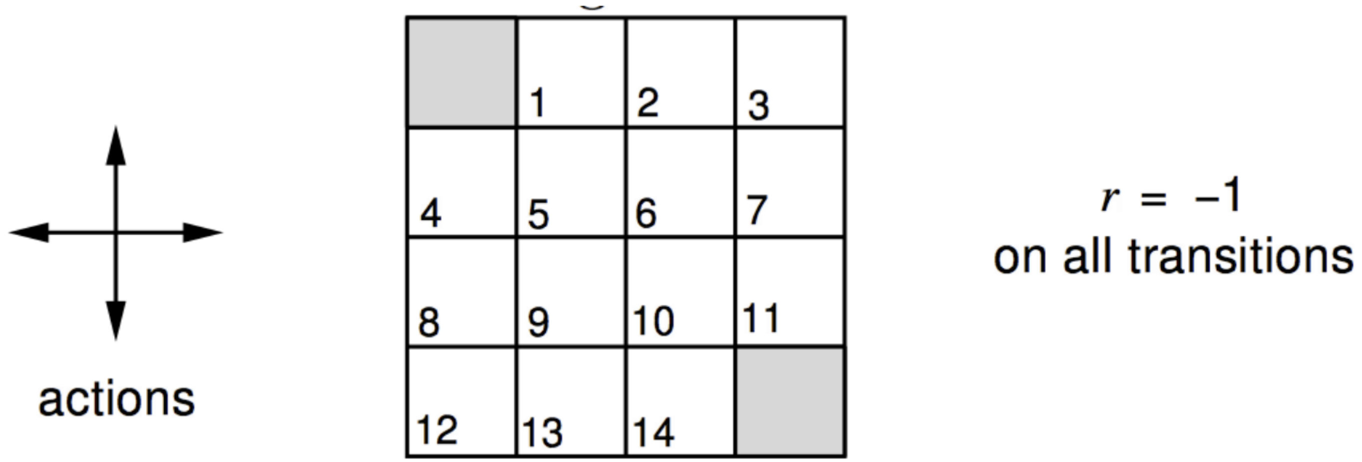
  $\pi' = \text{greedy}(v_\pi)$

- Converting back and forth between prediction and control
- Start with random policy, eval it, improve value, improve policy

Value Iteration

- Similar to Policy Iteration but start with random value function, recursively improve it
- Exercise to figure out equations if you start with random value instead of policy

Put it together



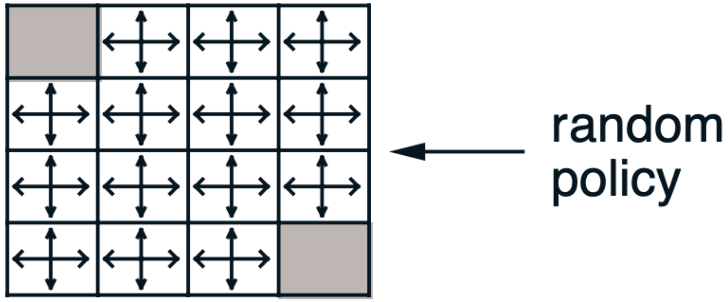
- Undiscounted episodic MDP ($\gamma = 1$)
- Nonterminal states 1, …, 14
- One terminal state (shown twice as shaded squares)
- Actions leading out of the grid leave state unchanged
- Reward is –1 until the terminal state is reached
- Agent follows uniform random policy $\pi(n|\cdot) = \pi(e|\cdot) = \pi(s|\cdot) = \pi(w|\cdot) = 0.25$

v_k for the Random Policy

Greedy Policy w.r.t. v_k

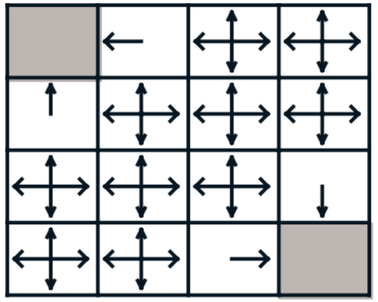
$k = 0$

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0



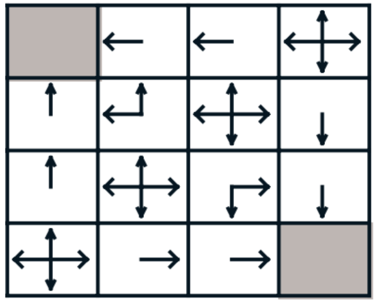
$k = 1$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0



$k = 2$

0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0



$k = 3$

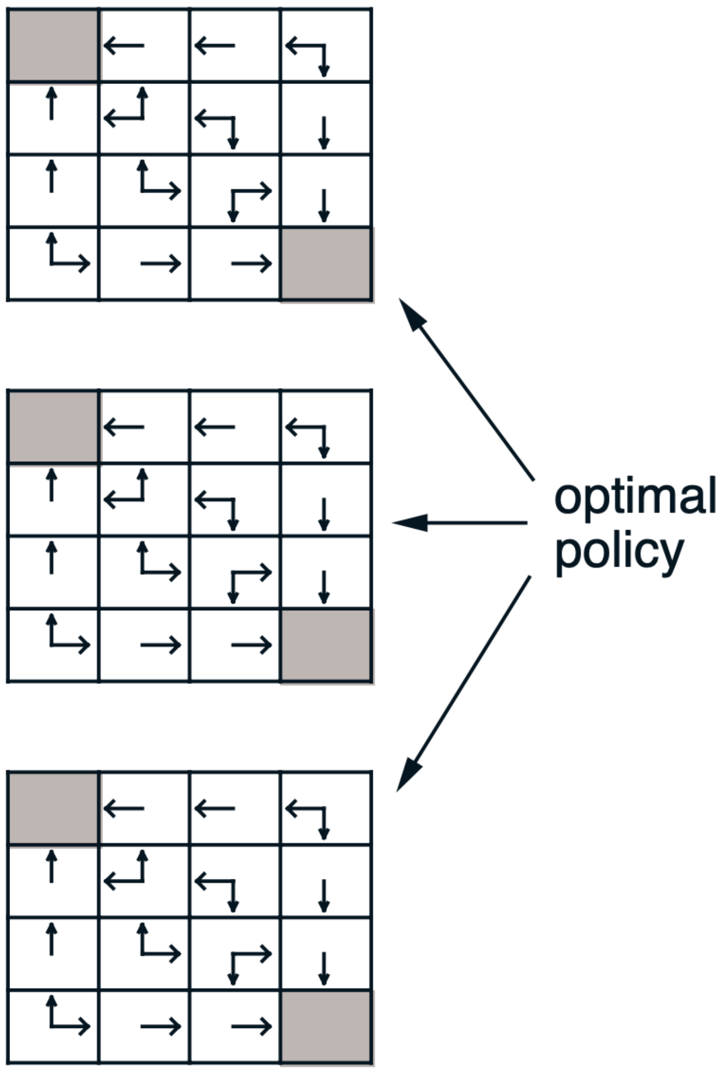
0.0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0.0

$k = 10$

0.0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0.0

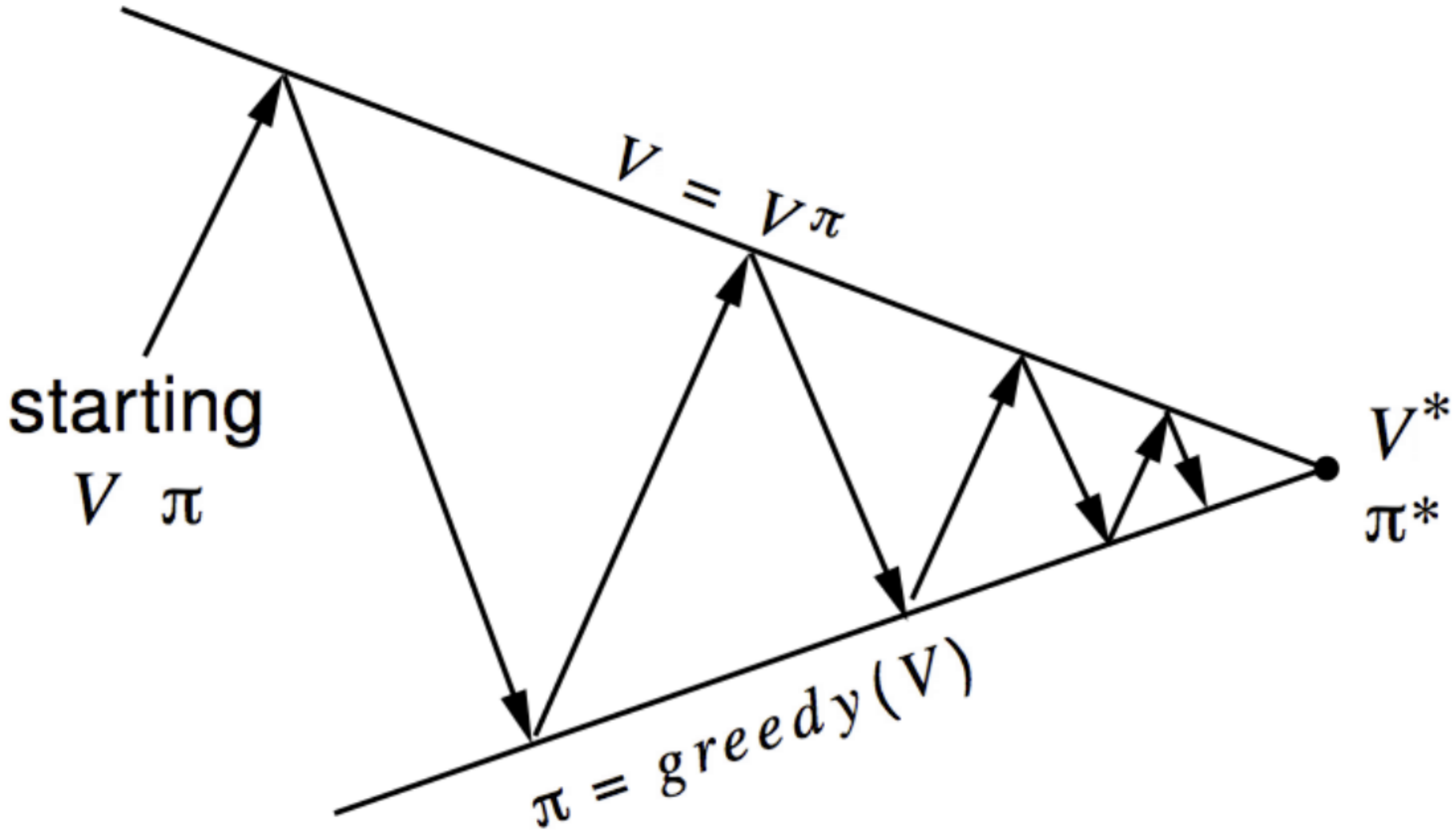
$k = \infty$

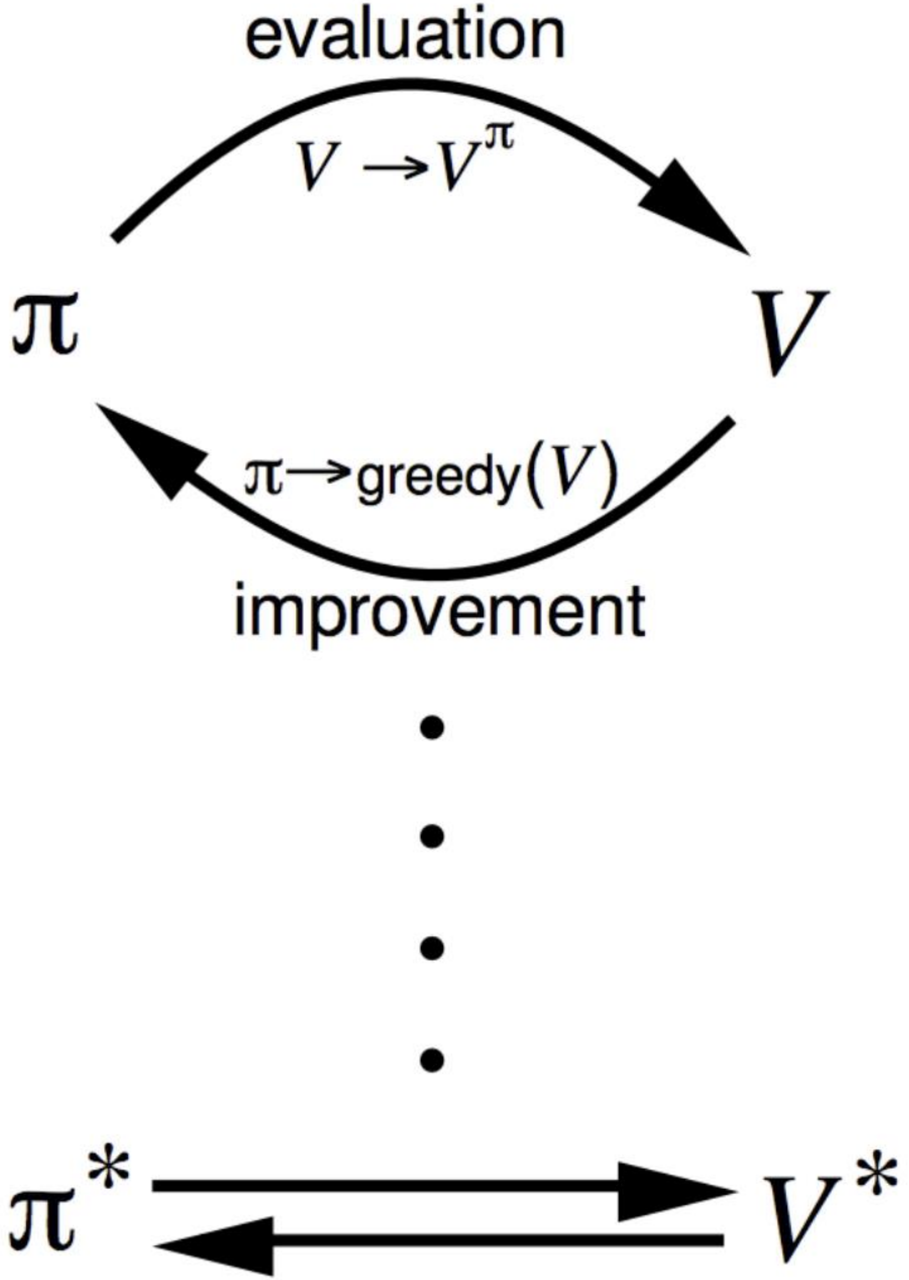
0.0	-14.	-20.	-22.
-14.	-18.	-20.	-20.
-20.	-20.	-18.	-14.
-22.	-20.	-14.	0.0



Generalized Policy Iteration

- Both are iterative versions of this





DP Limitations

- DP uses full-width backups
- For each backup Every successor state and action is considered
- Using knowledge of the MDP transitions and reward function DP is effective for medium-sized problems (millions of states)
- For large problems DP suffers Bellman’s curse of dimensionality
- Number of states $n = |S|$ grows exponentially with number of state variables Even one backup can be too expensive

Model Free RL via Sample Backups

- Model Free RL: optimize value of unknown MDP
- Using sample rewards and sample transitions <S, A, R’, S’>
  Instead of reward function R and transition dynamics T
- Advantages: Model-free: no advance knowledge of MDP required
  Breaks the curse of dimensionality through sampling
- Cost of backup is constant, independent of n = |S|

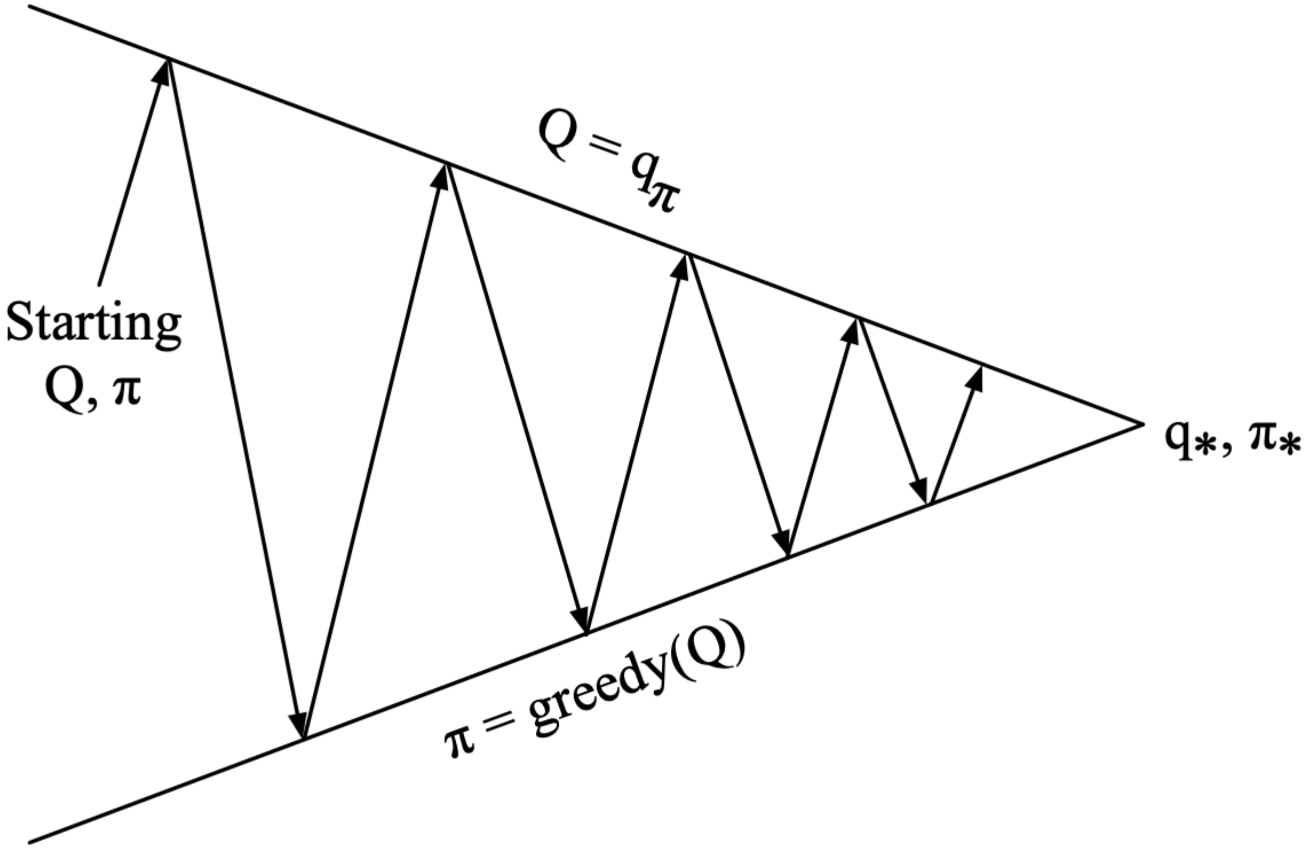
Experience Based Learning

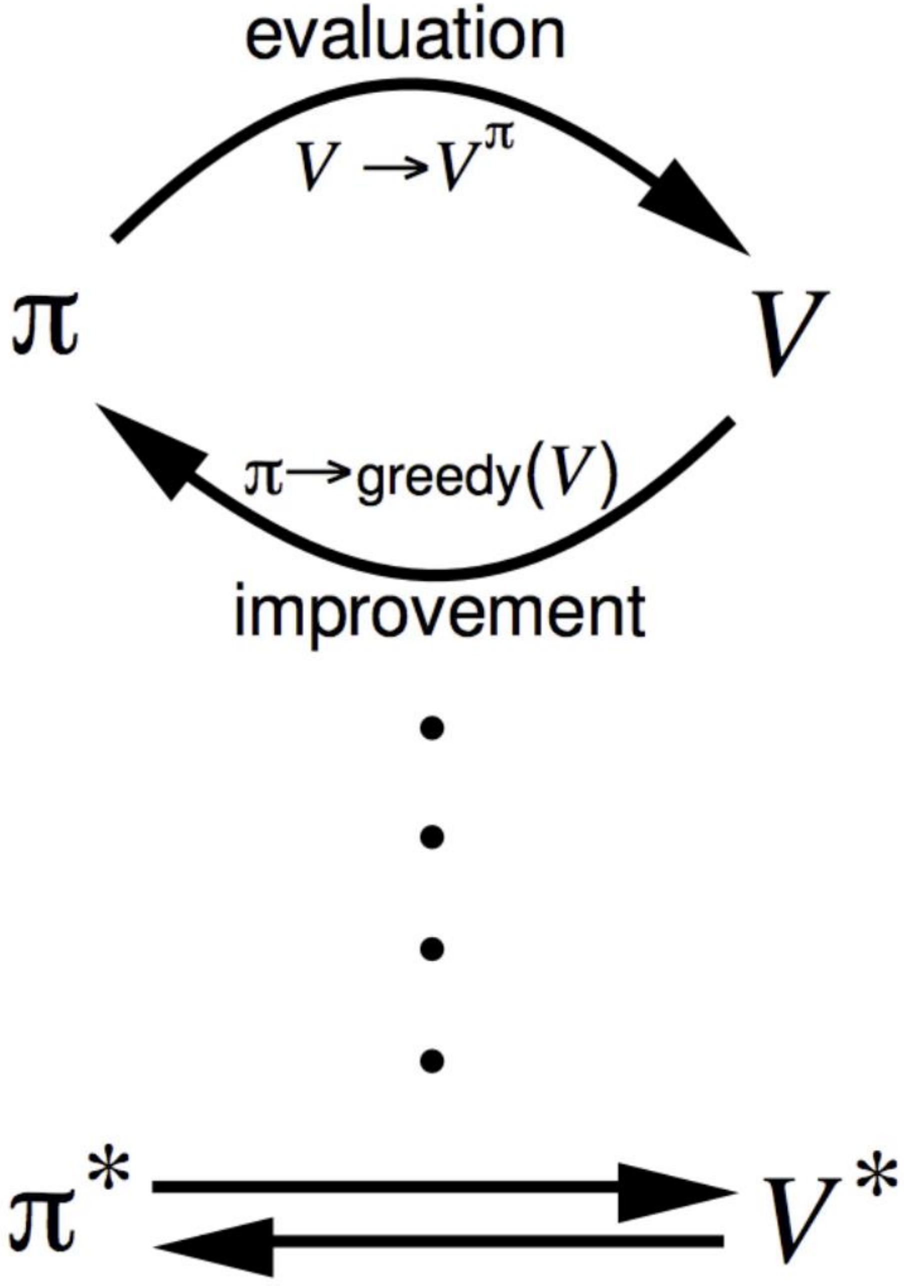
- Many real world problems are better suited to being solved by RL as opposed to DP based planning
- All the examples of agents we talked about first class
    - Robots in your home
    - Video games harder than tic tac toe
    - Language

Monte Carlo Control

- Greedy policy improvement over V(s) requires model of MDP
  $\pi'(s) = \mathrm{argmax}_{a \in A} R^a_s + \gamma \sum_{s' \in S} T^a_{ss'} V'(s')$
- Greedy policy improvement over Q(s, a) is model-free
  $\pi'(s) = \mathrm{argmax}_{a \in A} Q(s, a)$
- <u>Learn</u> this Q by function approximation using the experiences you've gathered by Monte Carlo sampling

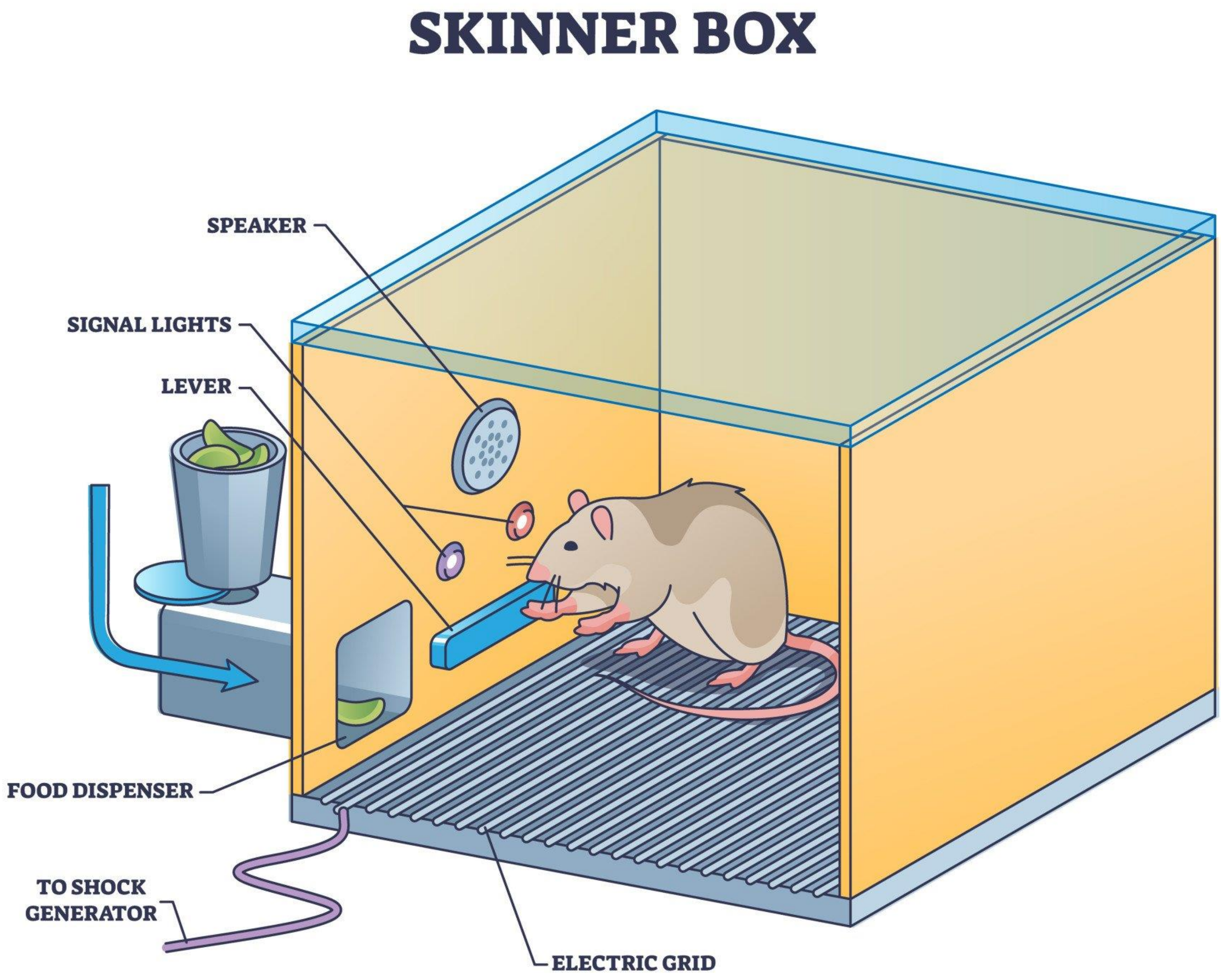
Generalized Policy Iteration
Monte Carlo Evaluation





Greedy Policy Improvement Limitations

- Greedy doesn't let you always explore all the actions you need



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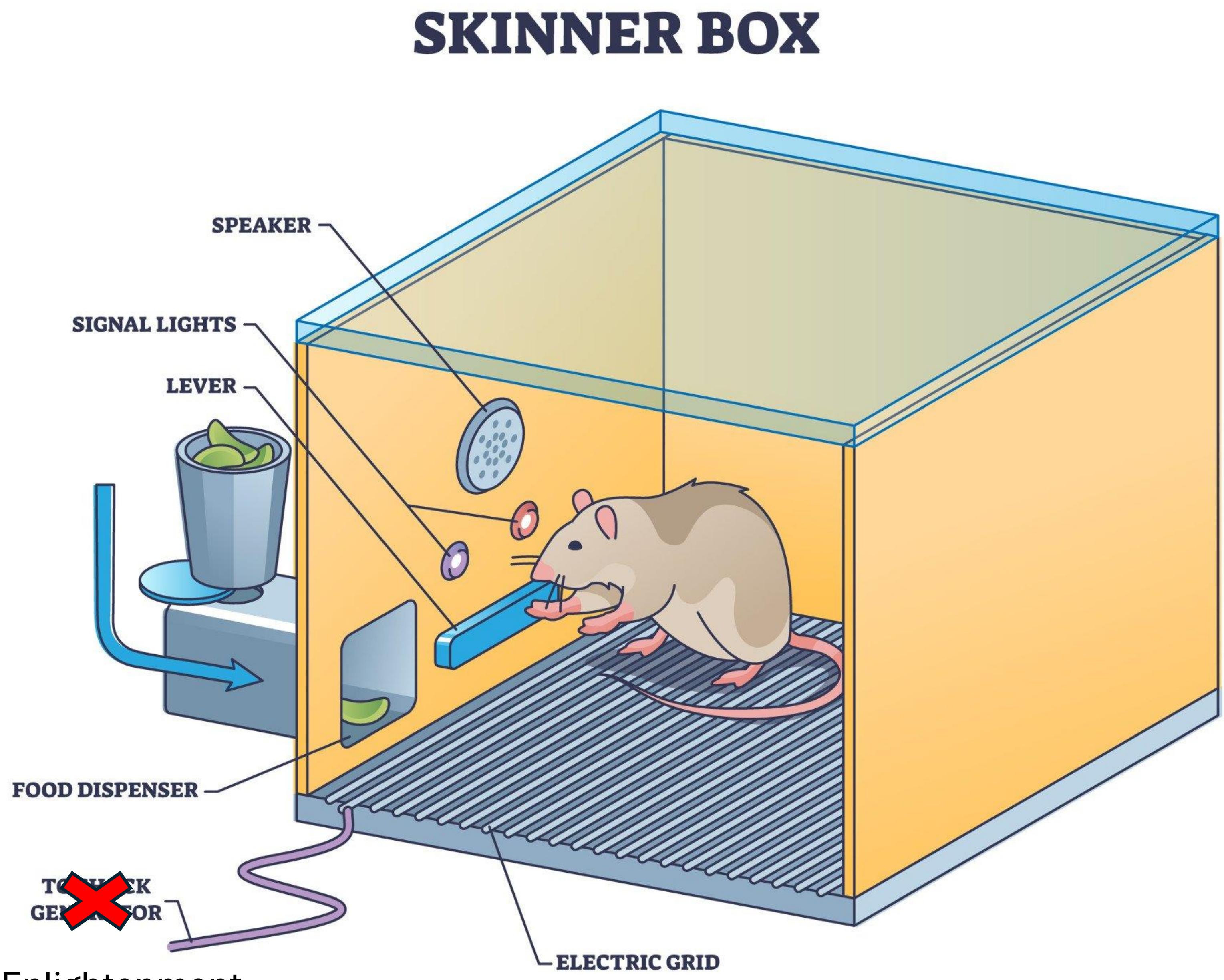


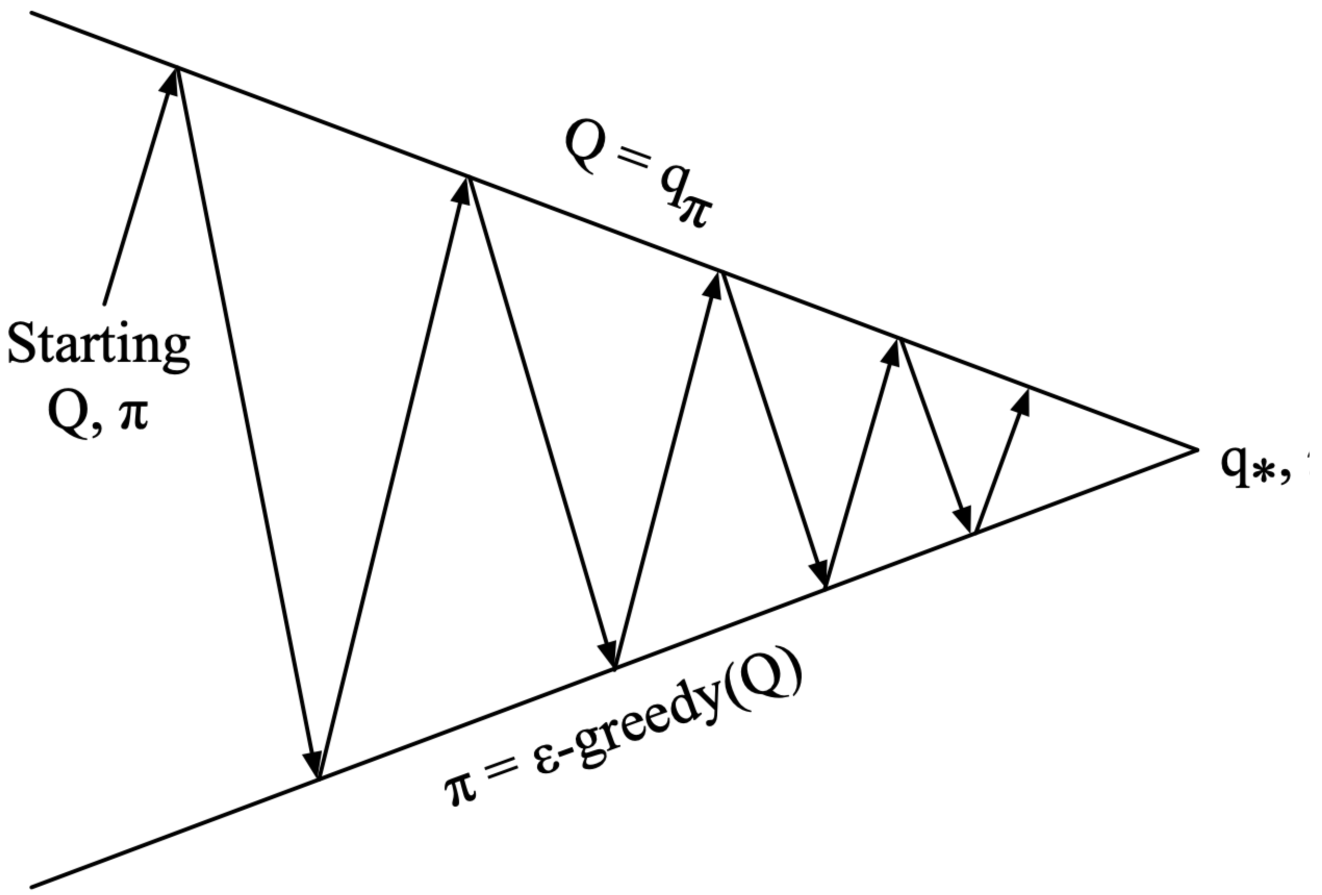
Enlightenment

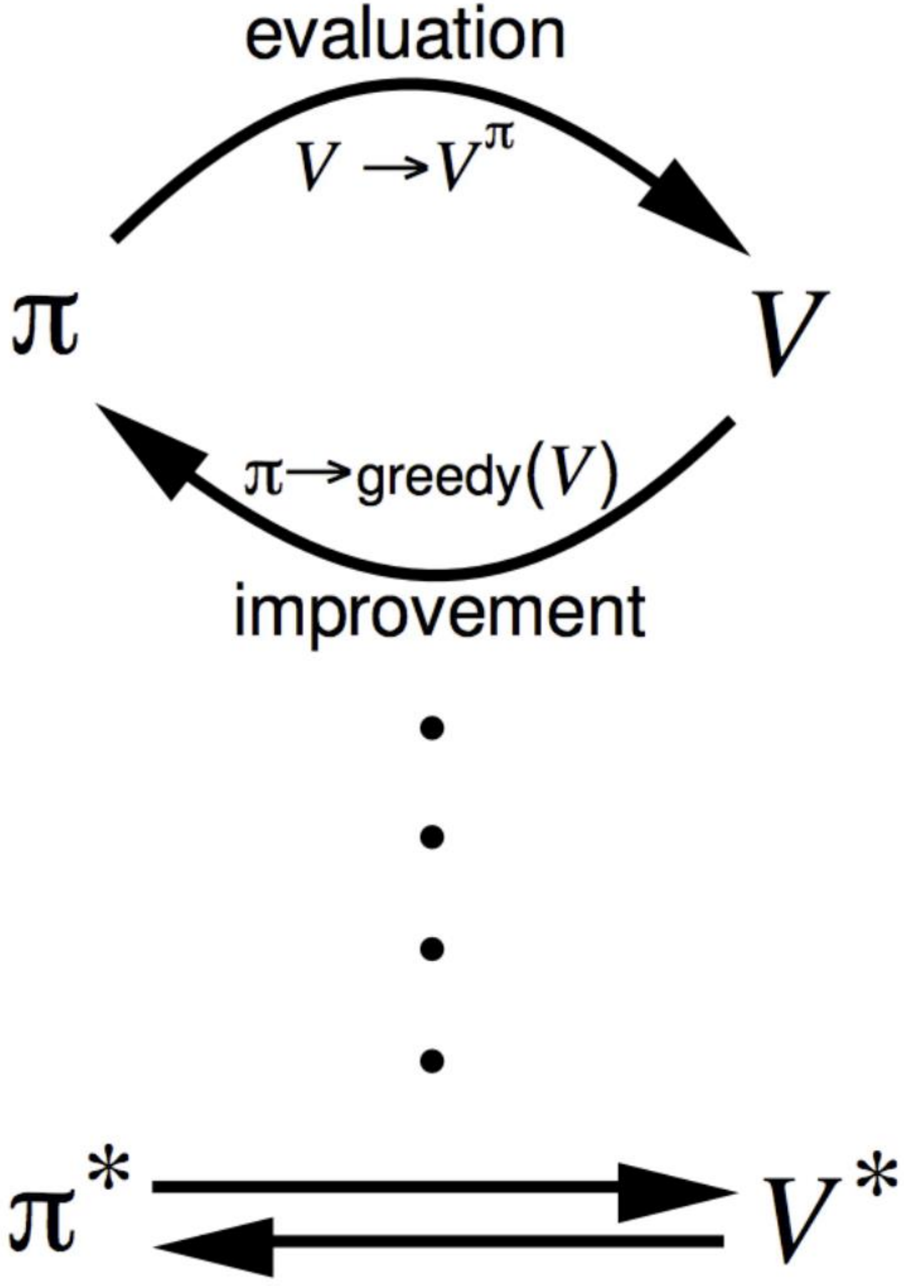
ε-greedy exploration

- Simplest idea for ensuring continual exploration
- All m actions are tried with non-zero probability
- With probability 1 − choose the greedy action
- With probability choose an action at random

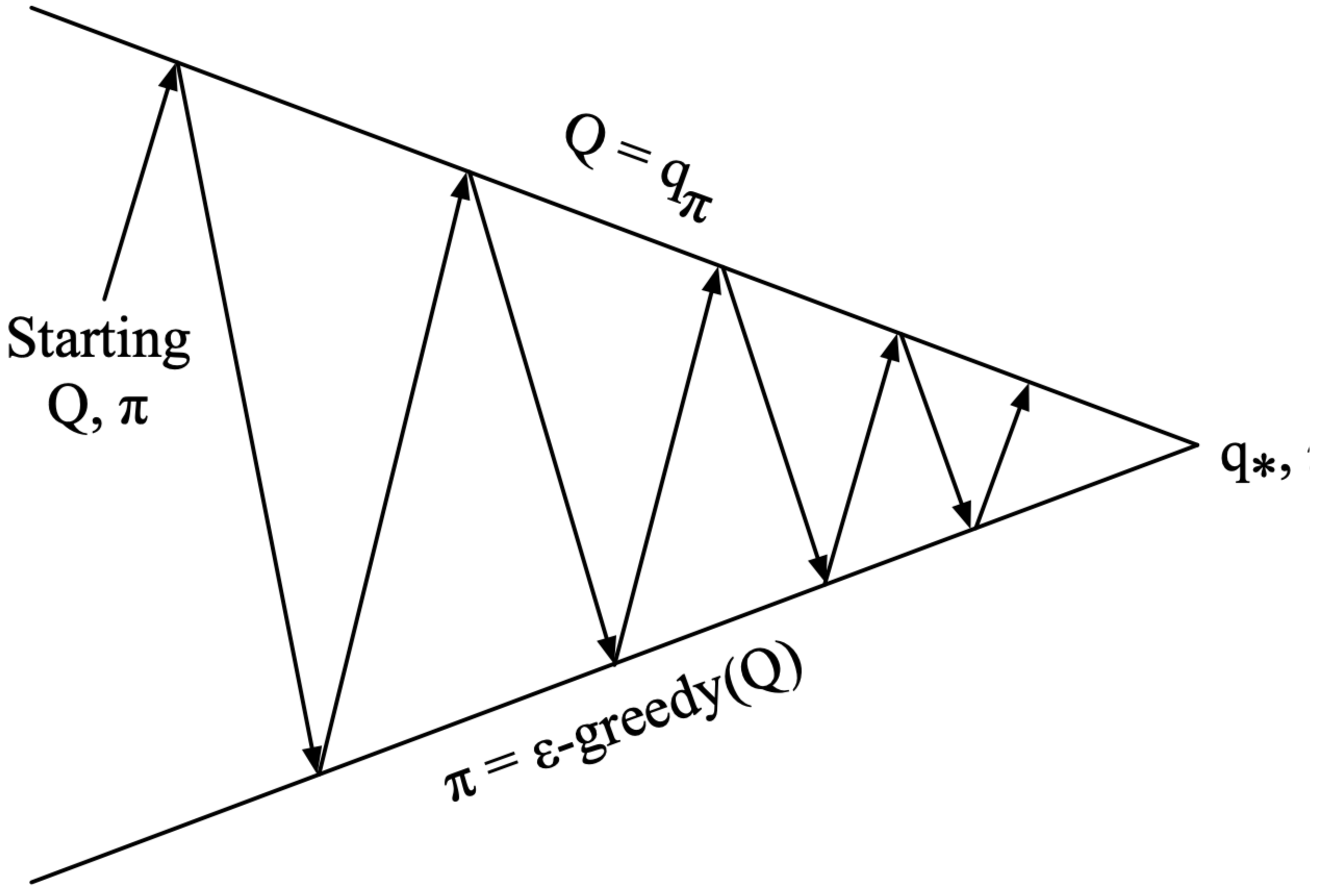
$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\ Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

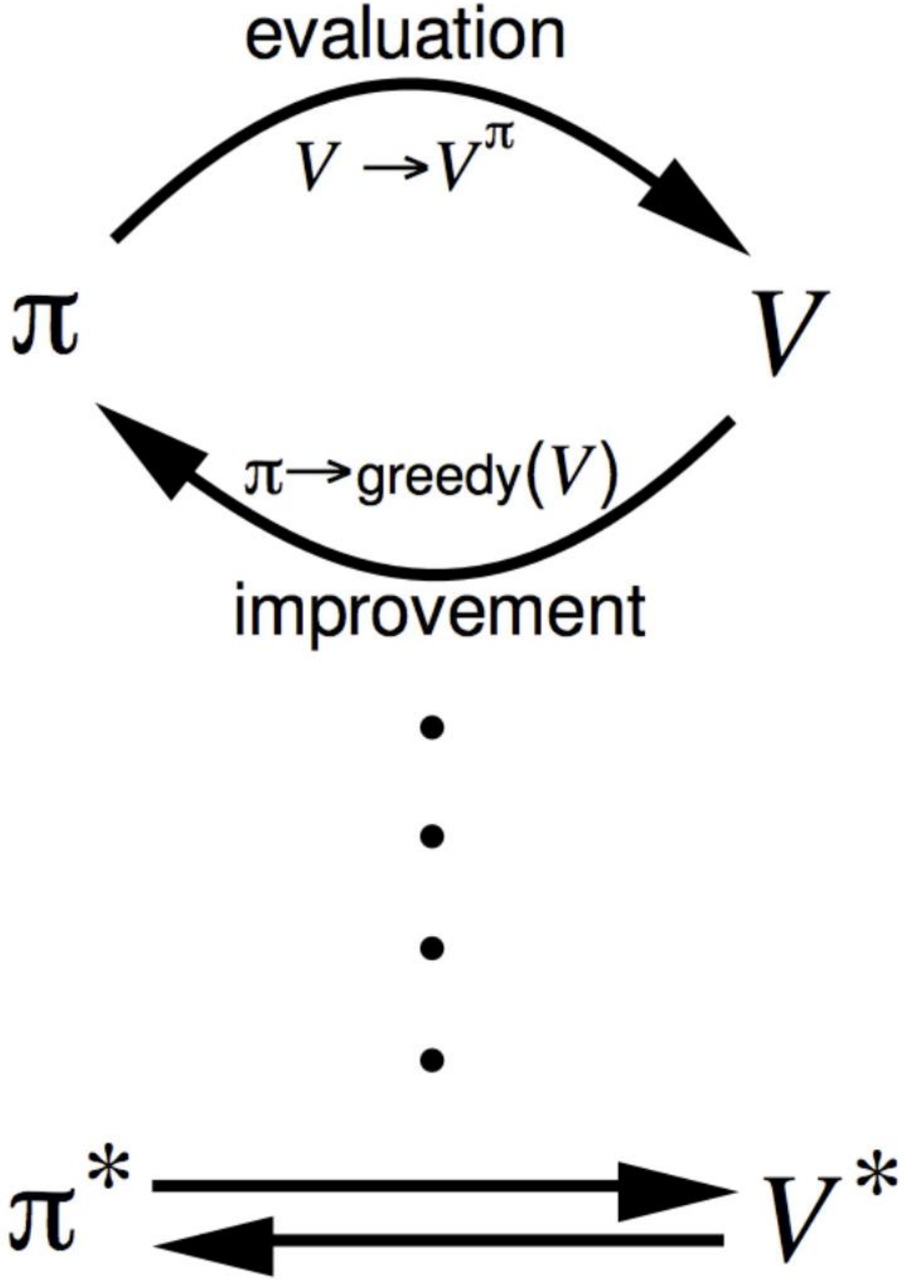
Generalized Policy Iteration
Monte Carlo Evaluation





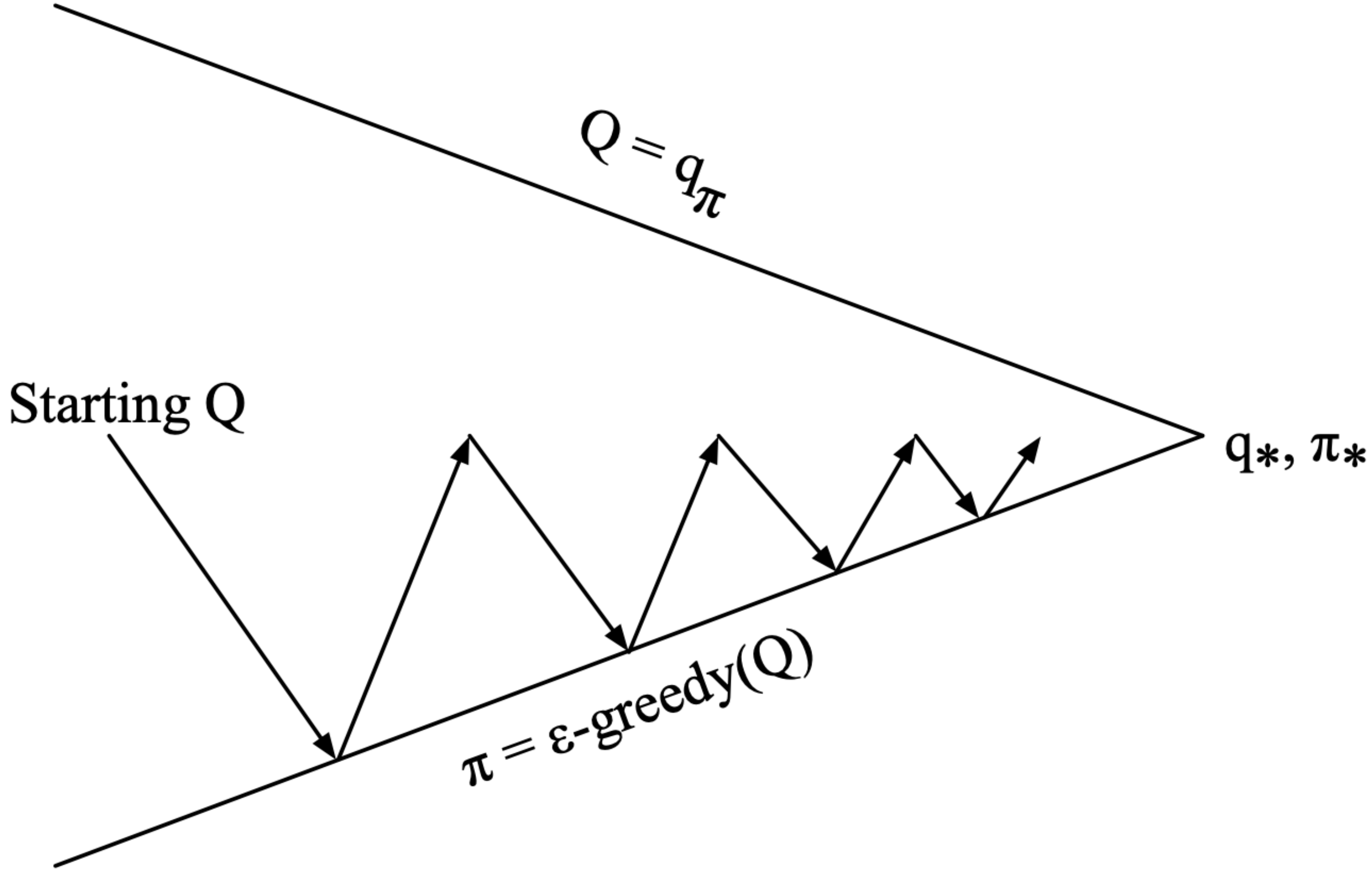
Generalized Policy Iteration
Monte Carlo Evaluation

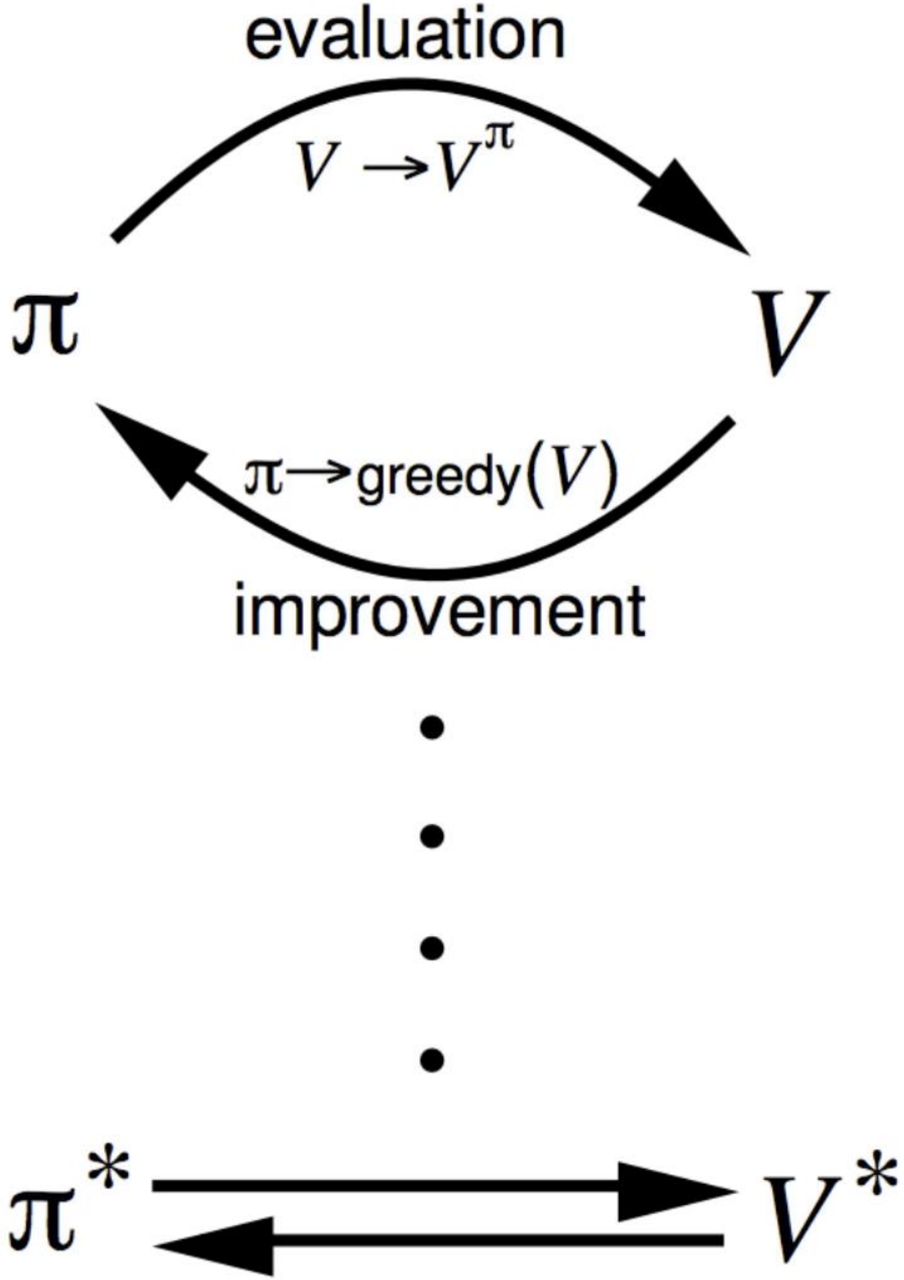




You can't fully evaluate the entire state space each time

Generalized Policy Iteration with Fn Approximation + Monte Carlo Eval





You can't fully evaluate the entire state space each time